What Do We Know about Estimating Government Spending Multipliers?

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September 2024

MOTIVATION

- What is the most accurate econometric approach of measuring government spending multipliers?

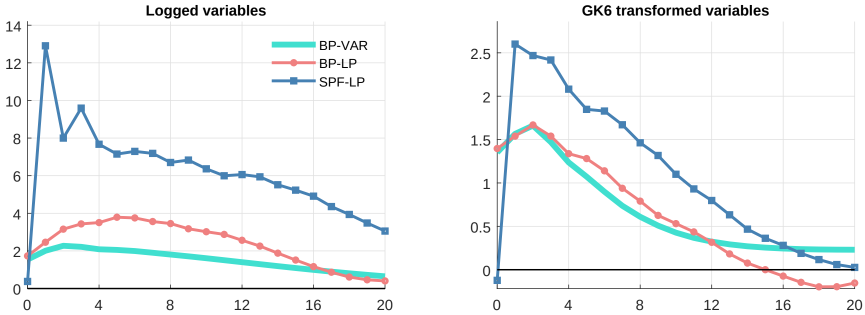
- In practice, there are at least three dimensions of time-series methods that lead to different multipliers

  (1) variable transformations, such as log or Gordon-Krenn (GK) transformations

  (2) which econometric model to use between vector autoregressions (VARs) and local projections (LPs)

  (3) how to identify government spending shocks

SIMILAR DATA, DIFFERENT MULTIPLIERS



Endogenous variables: G, T, Y, π, and R
Sample span: 1983:Q3$-$2007:Q4 / Lag length: 4

Government spending shocks are identified either using the method as in Blanchard and Perotti (2002, BP) or by relying on the SPF dataset.

Literature: Variable Transformations

- Owyang et al. (2013, AER)
  - log-transformations of the model's endogenous variables when calculating government spending multipliers tend to produce an upward bias
  - propose to use the transformation of variables as in Gordon and Krenn (2010)
  - e.g., Barro and Redlick (2011, QJE) and Ramey and Zubairy (2018, JPE)

LITERATURE: VARS VS. LPS

- Meier (2005)
  - horse racing between LP and VAR by employing the DSGE model of Smets & Wouters (2003, JEEA) as the true DGP
  - do not find evidence that LPs perform any better than VARs
- Plagborg-Møller and Wolf (2021, Econometrica)
  - LPs and VARs are conceptually identical methodologies under infinite lag
  - when the lag ℓ is finite, they have exactly the same impulse response estimates up to horizon ℓ
- Li, Plagborg-Møller and Wolf (2024)
  - LP estimators are associated with lower bias than VARs
  - however, LP estimators tend to have larger standard errors over medium to longer horizons

Literature: Shock Identification

- Blanchard and Perotti (2002, QJE)
  - use the institutional structure, including legislative and implementation lags, to identify these exogenous changes
- Forni and Gambetti (2016, JIE)
  - rely on economic agents forecasts of government spending by utilizing the Survey of Professional Forecasters (SPF) dataset

What We Do

(1) Employ a DSGE model as the true DGP

- take the new Keynesian DSGE model by Leeper, Traum and Walker (2017, AER)
- estimate the model's parameters with US time series of 1983:Q3–2007:Q4 (Bayesian inference)
- using the median of the estimated parameters, calculate the model-implied G multipliers and the sequence of G shocks

(2) Recover G multipliers by employing conventional econometric specifications in the existing literature

- log-transformation of the model's variables / Gordon-Krenn (GK) transformation (Gordon and Krenn, 2010)
- recursive VARs / LPs with shocks identified in various methods / LP with instrument variables (LP-IV)

- Compare G multipliers from (2) to those from (1)

What We Find

- The use of log-transformed variables is likely to result in an overestimation of the G multipliers for both VARs and LPs
  - the Gordon-Krenn transformation substantially reduces the overestimation of multipliers
- Among the GK-transformed specifications:
  - in the longer run, after about 12 quarters, recursive VARs tend to yield more accurate multipliers compared to LPs
    - but with the short-run overestimation of the G multipliers
  - if the true G shock sequence is known, LP using the true shock produces the most accurate short-run multipliers
  - if not, either a recurive VAR model or an LP using the shock sequence from a recursive VAR model can serve as a solid alternative

Theoretical Model

Model: Summary

Leeper, Traum and Walker (2017)

1. forward-looking, optimizing agents & rule-of-thumb agents
2. utility from consumption and leisure
3. capital and labor inputs in production
4. monopolistic competition
5. nominal & real frictions
6. fiscal and monetary policy

Model: In Detail

1. Preferences
   - habit in consumption
   - hand-to-mouth agents
   - households' utility from composite consumption: private and public consumption can be complements or substitutes
2. Technology
   - adjustment costs in investment
   - capacity utilization
3. Market structure: Imperfect competition
   - monopolistic competition in products and labor markets
   - price and wage stickiness

MONETARY POLICY

MP obeys a Taylor-type rule

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1-\rho_r)\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t\right) + \sigma_m \epsilon_t^m$$

where $\epsilon_t^m \sim N(0,1)$

- Nominal interest rate is set in response to fluctuations in output (Y_t) and inflation (π_t)

Fiscal Policy

- Government budget constraint:

$$B_t + \tau_t^K R_t^K K_{t-1} + \tau_t^L W_t L_t + \tau_t^C C_t = R_{t-1} B_{t-1} + G_t + Z_t$$

where

  - B_t: one-period nominal bonds
  - R_t^K: gross nominal rate of return from capital
  - τ_t^K, τ_t^L, τ_t^C: tax rates on capital income, labor income, consumption
  - G_t, Z_t: government spending and transfers

  - gov't spending and transfers are financed by proportional taxes levied against consumption, labor income, and capital returns, and by issuing one-period nominal debt

Fiscal Policy

- FP specification:

$$
\begin{aligned}
\hat{G}_t =& \rho_G \hat{G}_{t-1} - (1-\rho_G)\gamma_G \hat{s}^b_{t-1} + \sigma_G \epsilon^G_t \\
\hat{\tau}^K_t =& \rho_K \hat{\tau}^K_{t-1} + (1-\rho_K)\gamma_K \hat{s}^b_{t-1} + \sigma_K \epsilon^K_t \\
\hat{\tau}^L_t =& \rho_L \hat{\tau}^L_{t-1} + (1-\rho_L)\gamma_L \hat{s}^b_{t-1} + \sigma_L \epsilon^L_t \\
\hat{\tau}^C_t =& \rho_C \hat{\tau}^C_{t-1} + \sigma_C \epsilon^C_t \\
\hat{Z}_t =& \rho_Z \hat{Z}_{t-1} - (1-\rho_Z)\gamma_Z \hat{s}^b_{t-1} + \sigma_Z \epsilon^Z_t
\end{aligned}
$$

  where

  - $s^b_{t-1} \equiv B_{t-1}/Y_{t-1}$ (lagged debt-to-GDP ratio)
  - $\epsilon^X_t \sim N(0,1)$ where $X = \{G, K, L, C, Z\}$

Exogenous Disturbances

11 exogenous shocks:

- Tastes & technology: AR(1)
  - technology / investment-specific / preference / price markup / wage markup
- Policy: *i.i.d.*
  - MP disturbance
  - five FP disturbances

Data for Estimation

- 11 observable variables

1. consumption growth	7. government debt growth
2. investment growth	8. consumption tax revenue growth
3. real wage growth	9. labor tax revenue growth
4. hours worked	10. capital tax revenue growth
5. CPI inflation	11. government spending growth
6. nominal interest rate	

- Quarterly data from 1983:Q3 to 2007:Q4
  - the sample ends prior to the zero lower bound period

Estimation: Bayesian Inference

- Priors for the non-policy parameters are similar to Smets and Wouters (2007, AER) and Leeper et al. (2017)
- Other parameters fixed at well-established values (e.g., $\beta = 0.99$, $\alpha = 0.4$, $\delta = 0.025$)
- Prior for the parameters in the fiscal policy specification are drawn from Leeper et al. (2017)
- Random-walk MH, 30,000 final draws from posteriors

Parameter Estimates

Parameter		Prior			Posterior	
		Fnc.	Mean	Std.	Median	[5%, 95%]
Preference						
θ	Habit formation in consumption	B	0.5	0.2	0.96	[0.84, 0.98]
μ	Fraction of rule-of-thumb consumers	B	0.3	0.1	0.02	[0.01, 0.04]
α_G	Substitutability of private/public consumption	U	0	1.01	−0.11	[−0.42, 0.23]
Monetary Policy						
ρ_r	Lagged interest rate response	B	0.5	0.2	0.81	[0.74, 0.86]
ϕ_π	Interest rate response to inflation	N	1.5	0.2	1.40	[1.07, 1.73]
ϕ_y	Interest rate response to output	N	0.125	0.05	0.16	[0.11, 0.22]
Fiscal Policy						
γ_K	Capital tax response to debt/GDP	N	0.15	0.1	0.19	[0.04, 0.33]
γ_L	Labor tax response to debt/GDP	N	0.15	0.1	0.16	[0.05, 0.27]
γ_G	Gov't spending response to debt/GDP	N	0.15	0.1	0.26	[0.09, 0.41]
γ_Z	Transfers response to debt/GDP	N	0.15	0.1	0.09	[−0.04, 0.23]

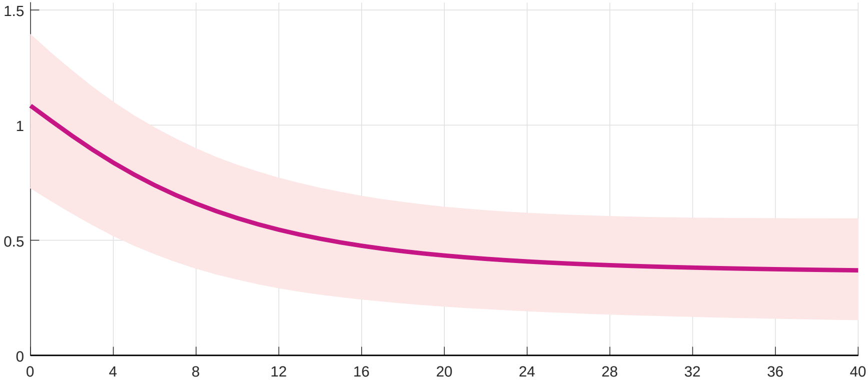
Gov't Spending Multiplier: Definition

1. Present Value Multiplier:

$$\text{Present Value Multiplier } (Q) = \frac{\sum_{t=0}^{Q} E_t \left(\prod_{i=0}^{Q} R_{t+i}^{-1} \right) \Delta Y_{t+Q}}{\sum_{t=0}^{Q} E_t \left(\prod_{i=0}^{Q} R_{t+i}^{-1} \right) \Delta G_{t+Q}}$$

2. Impact Multiplier: $Q = 0$

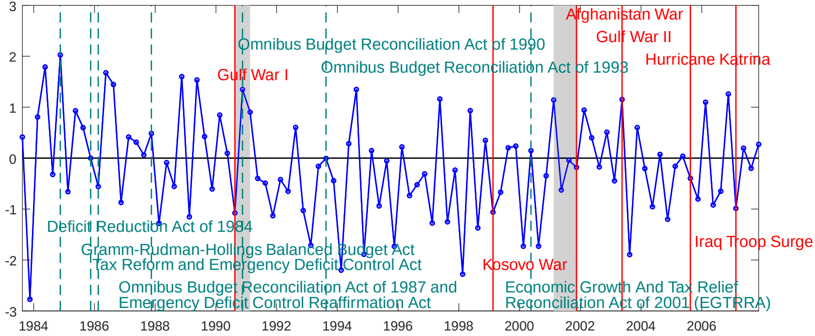
Model-based Multiplier Estimates



Present-value multipliers; Solid: Median; Shaded: [5%, 95%]

Model-based G Shock Sequences

Kalman-smoothed G shock sequence evaluated at the median param. estimates



Red vertical lines: War dates

Teal vertical lines: Fiscal acts or news about them

Econometric Methodologies

VAR and LP in the Literature

(1) Recursive VAR

- Blanchard and Perotti (2002)
  - 3 variables: G, T, Y
  - logged and quadratic detrended
- Perotti (2005), Caldara and Kamps (2008)
  - 5 variables: G, Y, π, T, R
  - logged and linearly detrended

Log Transformation

- The log(G)–log(Y) setup yields elasticities of Y w.r.t G
  - need an ex-post conversion factor, $\bar{Y}/\bar{G}$, to convert them into currency-valued multipliers

$$\text{Gov't spending multiplier } (k) = \underbrace{\frac{\Delta \log(Y_k)}{\Delta \log(G_k)}}_{\text{ratio between the irfs}} \frac{\bar{Y}}{\bar{G}} = \frac{\Delta Y_k / Y_k}{\Delta G_k / G_k} \frac{\bar{Y}}{\bar{G}} \approx \frac{\Delta Y_k}{\Delta G_k} \tag{1}$$

- Owyang, Ramey and Zubairy (2013): log transformations tend to produce an upward bias in multiplier estimates

Local Projection (LP)

- A simple linear LP specification:

$$z_{t+h} = \alpha_h + \beta_h S_t + \Gamma X_t + \varepsilon_{t+h} \tag{2}$$

  - z_{t+h}: outcome variable at horizon h
  - S_t: exogenous shock
  - X_t: control variables including the following
    - lagged z_t
    - exogenous variables
    - deterministic time trends

- Impulse response at horizon h

$$IR(h) \equiv E(z_{t+h}|S_t = s + \delta, X_t) - E(z_{t+h}|S_t = s, X_t) = \beta_h \delta$$

  - s: baseline (e.g., $s = 0$); δ: size of the shock

- Estimate "local" effects at each horizon $h \Longrightarrow$ robust to model misspecification relative to VARs

VAR and LP in the Literature

(2) LP (Jordá, 2005, AER)

- Auerbach and Gorodnichenko (2012; 2013, AER)
  - estimate state-dependent fiscal multipliers (panel LP)
  - forecast error shock
  - AG12: logged
  - AG13: Y_{t+h} and G_{t+h} are transformed into $(Y_{t+h} - Y_{t-1})/Y_{t-1}$ and $(G_{t+h} - G_{t-1})/Y_{t-1}$, respectively
- Owyang, Ramey and Zubairy (2013)
  - estimate state-dependent fiscal multipliers
  - self-constructed G news shock
  - Y_{t+h} and G_{t+h} are transformed into $(Y_{t+h} - Y_{t-1})/Y_{t-1}$ and $(G_{t+h} - G_{t-1})/Y_{t-1}$, respectively

VAR AND LP IN THE LITERATURE

(3) LP with instrument variable: Ramey and Zubairy (2018)

- First work in estimating fiscal multipliers using LP-IV
- LP-IV specification

$$\sum_{j=0}^{h} Y_{t+j} = \gamma_h + \phi_h(L) Z_{t-1} + m_h \sum_{j=0}^{h} G_{t+j} + \varepsilon_{t+h} \quad \text{for} \quad h = 0, 1, 2, .. \tag{3}$$

  where
  - Z is a vector of control variables
  - take into account the extended military spending news shock and BP shock as instruments for $\sum_{j=0}^{h} G_{t+j}$
- Y_t and G_t are Gordon-Krenn (GK) transformed

GORDON-KRENN TRANSFORMATION

- GK transformations
  - divides each real NIPA series by an estimate of potential real GDP
  - e.g., Ramey and Zubairy's (2018) procedure is as follows:
    1. regress logged real GDP on 6-th degree polynomial (GK6)
    2. take the exponential of the fitted value of the regression
  - now the variables are in levels, so do not need to use the ex-post conversion factor, $\bar{Y}/\bar{G}$, in calculating multipliers

GK Transformation: Algebra

- Recall gov't spending multiplier (h) = $\frac{\Delta Y_h}{\Delta G_h}$
- Consider a LP model as follows:

$$\tilde{y}_{t+h} = \alpha_h^y + \beta_h^y \tilde{g}_t + \phi_h^y(L)x_{t-1} + \varepsilon_{t+h}^y, \tag{4}$$
$$\tilde{g}_{t+h} = \alpha_h^g + \beta_h^g \tilde{g}_t + \phi_h^g(L)x_{t-1} + \varepsilon_{t+h}^g, \tag{5}$$

where $\tilde{y}$ and $\tilde{g}$ are divided by the trend of Y (Y_p)

- then

$$\beta_h^y = \frac{\Delta y_{t+h}}{\Delta g_t} = \frac{\Delta Y_{t+h}/Y_p}{\Delta G_t/Y_p}, \tag{6}$$
$$\beta_h^g = \frac{\Delta g_{t+h}}{\Delta g_t} = \frac{\Delta G_{t+h}/Y_p}{\Delta G_t/Y_p}. \tag{7}$$

GK Transformation: Algebra

- Define cumulative multiplier at horizon h:

$$m_h \equiv \frac{\sum_{k=0}^{h} \Delta Y_{t+k}}{\sum_{k=0}^{h} \Delta G_{t+k}} = \frac{\sum_{k=0}^{h} \beta_k^y}{\sum_{k=0}^{h} \beta_k^g} \tag{8}$$

- Equation (8) demonstrates that the ex-post conversion factor is not involved in calculating multipliers

VAR and LP Specifications in This Paper

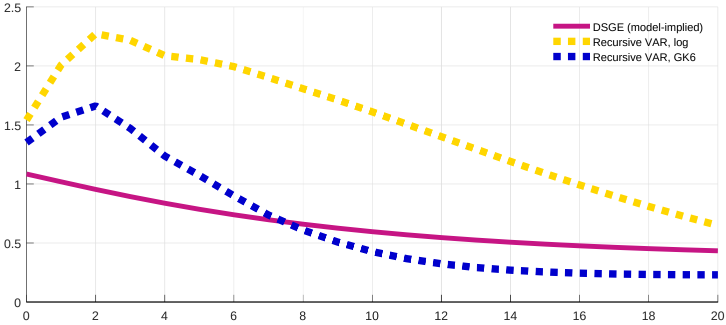
- 5-variable recursive VAR (Caldara and Kamps, 2008)
  - model's endogenous variables: G, Y, π, T, R
  - these variables are either log- or GK6-transformed

- 5-variable LP (Plagborg-Møller and Wolf, 2021)
  - model's endogenous variables are identical to the VAR
  - either log- or GK6-transformed
  - the LP model in equation:

$$Y_{t+h} = \alpha_h + \beta_h S_t + \phi_h(L) Z_{t-1} + \varepsilon_{t+h}, \quad h = 0, 1, 2, \ldots$$

where the lag of the control variables is set to be 4 ($L = 4$) and S_t uses various measures of changes in gov't spending

DSGE vs. VAR Multipliers

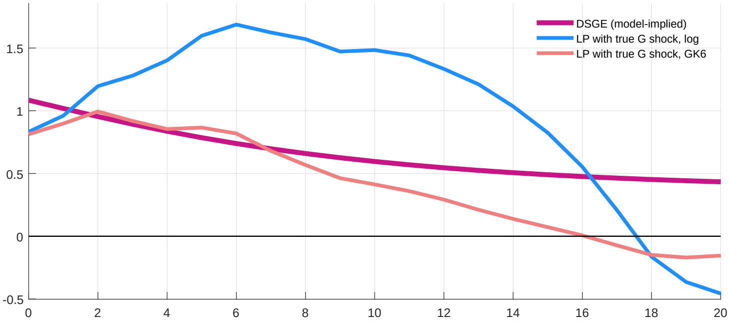


Pink: DSGE; Gold: Recursive VAR, log; Blue: Recursive VAR, GK6

Issue of the Ex-post Conversion Factor

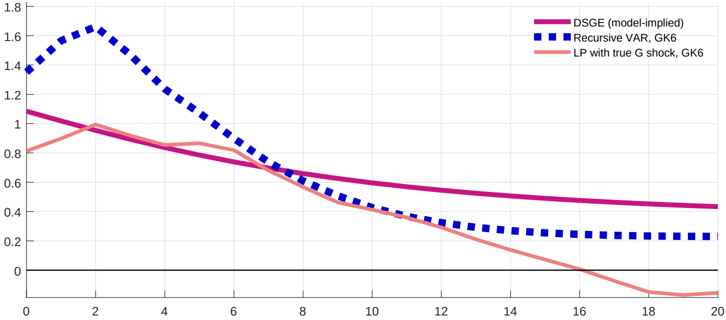
- $\bar{Y}/\bar{G} \approx 8$ in Ramey and Zubairy (2018)
- The ex-post conversion factor in our data turns out to be quite comparable to Ramey and Zubairy (2018)
  - the VAR in log can produce large multipliers

DSGE vs. LP with True G Shock



Pink: DSGE; Light blue: LP with true G shock, log;
Coral: LP with true G shock, GK6

DSGE, VAR AND LP W/ TRUE G SHOCK: GK6



Pink: DSGE; Blue: Recursive VAR, GK6 (RMSE: 0.31);

Coral: LP with true G shock, GK6 (RMSE: 0.33)

LP with BP or FE Shocks

- In practice, the true shock sequence is unlikely to be known to practitioners

- More feasible approaches employed in the existing LP literature involve using one of the following variables

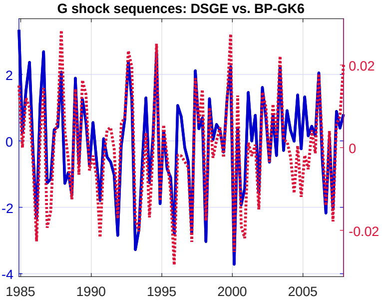
  (1) Blanchard-Perotti (BP) shock identified from SVARs
    - identical to the recursive VAR in our setup

  (2) forecast error (FE) shock is constructed as follows:
    - define forecast error as $fe_t = G_t - E_{t-1}(G_t)$ where E_t denotes expectations associated with the DSGE model
    - as in Auerbach & Gorodnichenko (2013), regress fe_t on the contemporaneous and lagged values of $[\Delta G, \Delta Y, \pi, \Delta T, R]$, and FE shock is defined as the residual of the regression

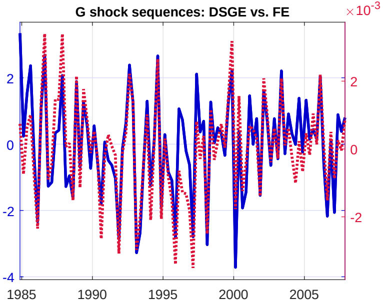
- Only specifications transformed by GK6 are considered

G Shocks: Comparison

Correlation coefficient $= 0.88$

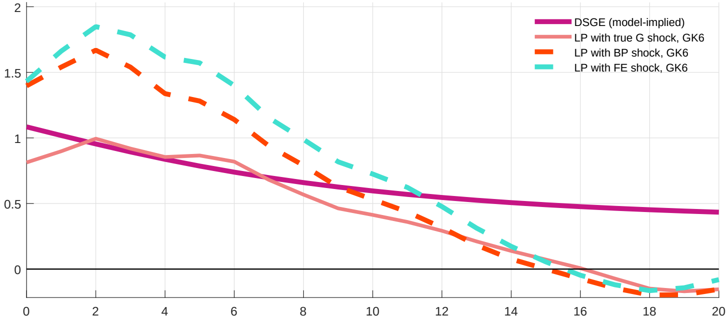


Correlation coefficient $= 0.86$



Blue: DSGE; Red: Econometrically recovered

DSGE AND LP WITH VARIOUS G SHOCKS



Pink: DSGE; Coral: LP with true G shock, GK6 (RMSE: 0.33);

Orange: LP with BP shock, GK6 (RMSE: 0.47);

Mint: LP with FE shock, GK6 (RMSE: 0.55)

LP-IV

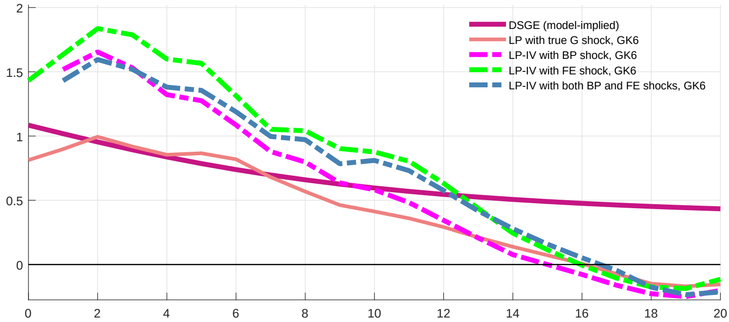
- Recall Ramey and Zubairy's (2018) specification:

$$\sum_{j=0}^{h} Y_{t+j} = \gamma_h + \phi_h(L) Z_{t-1} + m_h \sum_{j=0}^{h} G_{t+j} + \varepsilon_{t+h}, \quad h = 0, 1, 2, \ldots$$

  where
  - all the model's NIPA variables are GK transformed
  - m_h's can be interpreted as cumulative multipliers

- Three candidates as an instrument for $\Sigma_{j=0}^{h} G_{t+j}$
  1. BP shock
  2. FE shock
  3. both 1 and 2

DSGE AND LP-IV



Pink: DSGE; Coral: LP with true G shock, GK6 (RMSE: 0.33);
Magenta: LP-IV with BP shock, GK6 (RMSE: 0.47);
Lime: LP-IV with FE shock, GK6 (RMSE: 0.54);
Dark blue: LP-IV with both shocks, GK6 (RMSE: 0.45)

LP-IV: F-Statistics

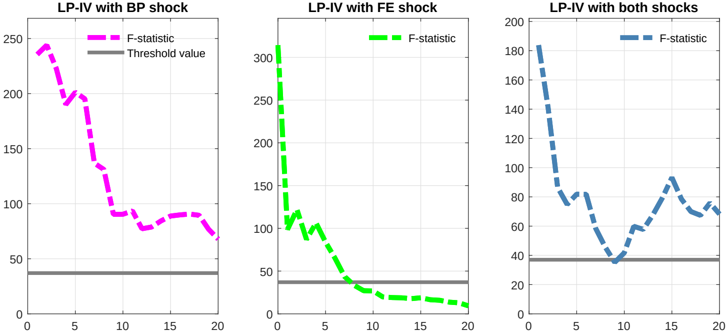
- Suppose that I_t represents one of these external series
- Then this series is a valid instrument for identifying the true shock $\varepsilon_{1,t}$ if the following two conditions hold:

  (1) $E[I_t \varepsilon_{1,t}] \neq 0$: *relevance* condition

  (2) $E[I_t \varepsilon_{i,t}] = 0$ for all $i \neq 1$: *exogeneity* condition

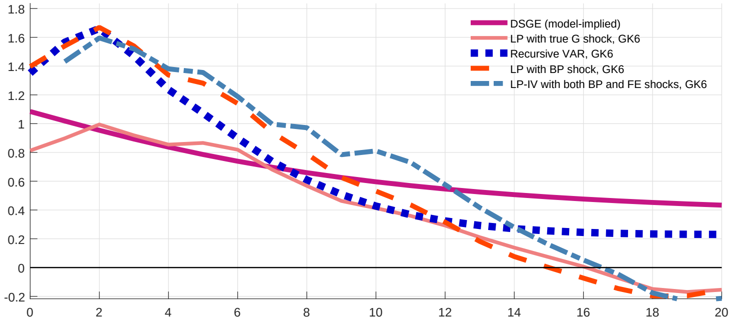
  - condition (1) can be checked by using the F-statistic as in Olea and Pflueger (2013, JBES)

LP-IV: F-Statistics



Horizontal line: Threshold value of 37 in Olea and Pflueger (2013)

OVERALL



Pink: DSGE; Coral: LP with true G shock, GK6 (RMSE: 0.33);

Blue: Recursive VAR, GK6 (RMSE: 0.31);

Orange: LP with BP shock, GK6 (RMSE: 0.47);

Dark blue: LP-IV with both shocks, GK6 (RMSE: 0.45)

OVERALL: RMSE

Specification	1−20 quarters	1−8 quarters	9−20 quarters
LP with the true G shock, log	0.69	0.69	0.69
LP with the true G shock, GK6	0.33 (2)	0.07 (1)	0.42
Recursive VAR, log	0.95	1.22	0.70
Recursive VAR, GK6	0.31 (1)	0.42 (2)	0.21 (1)
LP with the BP shock, GK6	0.47	0.49	0.45
LP with the FE shock, GK6	0.55	0.71	0.41
LP-IV with the BP shock, GK6	0.47	0.47	0.47
LP-IV with the FE shock, GK6	0.54	0.68	0.42
LP-IV with both the BP & FE shocks, GK6	0.45	0.50	0.41 (2)

Root mean squared errors (RMSE) associated with each econometric methodology. Rankings in terms of the RMSE statistics are reported in parentheses.

Key Takeaways

1. Using the GK transformation instead of the log transformation tends to reduce the upward bias introduced by econometric models such as VARs and LPs

2. The choice between VARs and LPs should be guided by the quality of the identified government spending shocks

   - if there is a strong candidate that effectively captures exogenous changes in government spending, using the shock sequence within an LP framework can help produce reliable estimates of government spending multipliers, particularly in the short run
   - if not, either a BP-identified VAR model or an LP using the shock sequence from a BP-identified VAR model can serve as a solid alternative